

Thermal and non-thermal leptogenesis in different neutrino mass models with tribimaximal mixings

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Abstract

In the present work we study both thermal and non-thermal leptogenesis in all neutrino mass models describing the presently available neutrino mass patterns. We consider the Majorana CP violating phases coming from right-handed Majorana mass matrices to estimate the baryon asymmetry of the universe, for different neutrino mass models namely degenerate, inverted hierarchical and normal hierarchical models, with tribimaximal mixings. Considering two possible diagonal forms of Dirac neutrino mass matrix as either charged lepton or up-quark mass matrix, the right-handed Majorana mass matrices are constructed from the light neutrino mass matrix through the inverse seesaw formula. Only the normal hierarchical model leads to the best predictions for baryon asymmetry of the universe, consistent with observations in both thermal and non-thermal leptogenesis scenario. The analysis though phenomenological may serve as an additional information in the discrimination among the presently available neutrino mass models.

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1 Introduction

The existence of heavy right-handed Majorana neutrinos in some of the left-right symmetric GUT models, not only gives small but non vanishing neutrino masses through the celebrated seesaw mechanism[1], it also plays an important role in explaining the baryon asymmetry of the universe [2] $Y_B = (6.1_{-0.2}^{+0.3}) 10^{-10}$. Such an asymmetry can be dynamically generated if the particle interaction rate and the expansion rate of the universe satisfy Sakharov's three famous conditions [3]. Majorana right-handed neutrinos satisfy the second condition i.e, C and CP violation as they can have an asymmetric decay to leptons and Higgs particles, and the process occurs at different rates for particles and antiparticles. The lepton asymmetry is then partially converted to baryon asymmetry through the non-perturbative electroweak sphaleron effects [4,5]. In such thermal leptogenesis the right-handed neutrinos can be generated thermally after inflation, if their masses are comparable to or below the reheating temperature $M_1 \leq T_R$. This allows high scale reheating temperature $T_R \geq 10^9$ GeV[6]. In non-thermal leptogenesis[7] it is possible to produce lepton asymmetry by using the low reheating temperature, where the right-handed neutrinos are produced through the direct non-thermal decays of the inflaton. This is particularly important for supersymmetric models where gravitino problem[8] can be avoided provided the reheating temperature after inflation is bounded from above in a certain way, namely $T_R \leq (10^6 - 10^7)$ GeV.

In order to calculate the baryon asymmetry from a given neutrino mass model, one usually starts with the light neutrino mass matrices m_{LL} and then relates it with the heavy Majorana neutrinos M_{RR} and the Dirac neutrino mass matrix m_{LR} through inverse seesaw mechanism in an elegant way. We consider the Dirac neutrino mass matrix m_{LR} as either the charged lepton mass matrix or up quark mass matrix for phenomenological analysis. The complex CP violating phases are usually derived from the MNS leptonic mixing matrix. In the present work we are interested to consider the complex Majorana phases which are derived from the right-handed Majorana mass matrix M_{RR} , in the estimation of baryon asymmetry of the universe. We wish to consider the left-handed light Majorana neutrino mass matrices m_{LL} which obey the $\mu - \tau$ symmetry[9] where tribimaximal mixings[10] are realised, for all possible patterns of neutrino masses, viz, degenerate, inverted hierarchical and normal hierarchical mass patterns. We first parametrise the light left-handed Majorana neutrino mass matrices which are subjected to

correct predictions of neutrino mass parameters and mixing angles. The calculation of baryon asymmetry of the universe in the light of thermal as well as non-thermal leptogenesis, may serve as an additional information to further discriminate the correct pattern of neutrino mass models and also shed light on the structure of Dirac neutrino mass matrix.

In section 2, we briefly mention the formalism for estimating the lepton asymmetry in thermal leptogenesis through out of the equilibrium decay of the heavy right-handed Majorana neutrinos, followed by numerical calculation and results. Section 3 is devoted to non-thermal leptogenesis and numerical predictions. Finally in section 4 we conclude with a summary and discussion. Important expressions related to m_{LL} which obey $\mu-\tau$ symmetry for three neutrino mass models, are relegated to Appendix A.

2 Baryon asymmetry of the universe in thermal leptogenesis

The canonical seesaw formula (known as type-I)[1] relates the left-handed Majorana neutrino mass matrix m_{LL} and heavy right handed Majorana mass matrix M_{RR} in a simple way

$$m_{LL} = -m_{LR}M_{RR}^{-1}m_{LR}^T \quad (1)$$

where m_{LR} is the Dirac neutrino mass matrix. For our calculation of lepton asymmetry, we consider the model[5,11] where the asymmetric decay of the lightest of the heavy right-handed Majorana neutrinos, is assumed. The physical Majorana neutrino N_R decays into two modes:

$$\begin{aligned} N_R &\rightarrow l_L + \phi^\dagger \\ &\rightarrow \bar{l}_L + \phi \end{aligned}$$

where l_L is the lepton and \bar{l}_L is the antilepton and the branching ratio for these two decay modes is likely to be different. The CP-asymmetry which is caused by the interference of tree level with one-loop corrections for the decays of lightest of heavy right-handed Majorana neutrino N_1 , is defined by [5,12]

$$\epsilon = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}$$

where $\Gamma = \Gamma(N_1 \rightarrow l_L \phi^\dagger)$ and $\bar{\Gamma} = \Gamma(N_1 \rightarrow \bar{l}_L \phi)$ are the decay rates. A perturbative calculation from the interference between tree level and vertex plus self energy diagrams, gives[13] the lepton asymmetry ϵ_1 for non-SUSY case as

$$\epsilon_i = -\frac{1}{8\pi} \frac{1}{(h^\dagger h)_{ii}} \sum_{j=2,3} \text{Im}[(h^\dagger h)_{ij}]^2 [f(\frac{M_j^2}{M_i^2}) + g(\frac{M_j^2}{M_i^2})] \quad (2)$$

where $f(x)$ and $g(x)$ represent the contributions from vertex and self-energy corrections respectively,

$$f(x) = \sqrt{x}[-1 + (x+1)\ln(1 + \frac{1}{x})],$$

$$g(x) = \frac{\sqrt{x}}{x-1}.$$

For hierarchical right-handed neutrino masses where x is large, we have the approximation[2], $f(x) + g(x) \simeq \frac{3}{2\sqrt{x}}$. This simplifies to

$$\epsilon_1 \simeq -\frac{3}{16\pi} \left[\frac{\text{Im}[(h^\dagger h)_{12}^2]}{(h^\dagger h)_{11}} \frac{M_1}{M_2} + \frac{\text{Im}[(h^\dagger h)_{13}^2]}{(h^\dagger h)_{11}} \frac{M_1}{M_3} \right] \quad (3)$$

where $h = m_{LR}/v$ is the Yukawa coupling of the Dirac neutrino mass matrix in the diagonal basis of M_{RR} . In term of light Majorana neutrino mass matrix m_{LL} , the above expression can be simplified to

$$\epsilon_1 \simeq -\frac{3}{16\pi} \frac{M_1}{(h^\dagger h)_{11} v^2} \text{Im}(h^\dagger m_{LL} h^*)_{11}.$$

For quasi-degenerate spectrum i.e., for $M_1 \simeq M_2 < M_3$ the asymmetry is largely enhanced by a resonance factor and in such situation, the lepton asymmetry is modified[14] to

$$\epsilon_1 \simeq \frac{1}{8\pi} \frac{\text{Im}[(h^\dagger h)_{12}^2]}{(h^\dagger h)_{11}} R \quad (4)$$

where

$$R = \frac{M_2^2(M_2^2 - M_1^2)}{(M_1^2 - M_2^2)^2 + \Gamma_2^2 M_1^2} \text{ and } \Gamma_2 = \frac{(h^\dagger h)_{22} M_2}{8\pi}$$

It can be noted that in case of SUSY, the functions $f(x)$ and $g(x)$ are given by $f(x) = \sqrt{x} \ln(1 + \frac{1}{x})$ and $g(x) = \frac{2\sqrt{x}}{x-1}$; and for large x one can have $f(x) + g(x) \simeq \frac{3}{\sqrt{x}}$. Therefore the factor $\frac{3}{8}$ will appear in place of $\frac{3}{16}$ in the expression of CP asymmetry[2].

The CP asymmetry parameter ϵ_1 is related to the leptonic asymmetry parameter through Y_L as

$$Y_L \equiv \frac{n_L - \bar{n}_L}{s} = \sum_i^3 \frac{\epsilon_i \kappa_i}{g_{*i}} \quad (5)$$

where n_L is the lepton number density, \bar{n}_L in the anti-lepton number density, s is the entropy density, κ_i is the dilution factor for the CP asymmetry ϵ_i , and g_{*i} is the effective number of degrees of freedom at temperature $T = M_i$. The baryon asymmetry n_B produced through the sphaleron transmutation of Y_L , while the quantum number $B - L$ remains conserved, is given by [15]

$$\frac{n_B}{s} = C Y_{B-L} = \frac{C}{C-1} Y_L \quad (6)$$

where

$$C = \frac{8N_F + 4N_H}{22N_F + 13N_H}. \quad (7)$$

Here N_F is the number of fermion families and N_H is the number of Higgs doublets. Since $s = 7.04n_\gamma$ the baryon number density over photon number density n_γ corresponds to the observed baryon asymmetry of the Universe[16],

$$Y_B^{SM} \equiv \left(\frac{n_B}{n_\gamma}\right)^{SM} \simeq d\kappa_1\epsilon_1 \quad (8)$$

where $d \simeq 0.98 \times 10^{-2}$ is used in the present calculation. In case of MSSM, there is no major numerical change with respect to the non-supersymmetric case in the estimation of baryon asymmetry. One expects approximate enhancement factor of about $\sqrt{2}(2\sqrt{2})$ for strong (weak) washout regime[2].

In the expression for baryon-to-photon ratio κ_1 describes the washout of the lepton asymmetry due to various lepton number violating processes. This efficiency factor (also known as dilution factor) mainly depends on the effective neutrino mass \tilde{m}_1

$$\tilde{m}_1 = \frac{(h^\dagger h)_{11} v^2}{M_1}$$

where v is the electroweak vev, $v = 174\text{GeV}$. For $10^{-2}\text{eV} < \tilde{m}_1 < 10^3\text{eV}$, the washout factor κ_1 can be well approximated by [12,17]

$$\kappa_1(\tilde{m}_1) = 0.3 \left[\frac{10^{-3}}{\tilde{m}_1} \right] \left[\log \frac{\tilde{m}_1}{10^{-3}} \right]^{-0.6}. \quad (9)$$

We adopt a single expression for κ_1 valid only for the given range of \tilde{m}_1 [17,18,19].

2.1 Numerical calculations and results

To compute the numerical results, we first choose the light left-handed Majorana neutrino mass matrix m_{LL} proposed in Appendix A. These mass matrices obey the $\mu - \tau$ symmetry [9] which guarantees the tribimaximal mixings [10]. The input parameters are fixed at the stage of predictions of neutrino mass parameters and mixings given in Table 1. These results are consistent with the recent data on neutrino oscillations.

For the calculation of baryon asymmetry, we then translate these mass matrices to M_{RR} via inversion of the seesaw formula, $M_{RR} = -m_{LR}^T m_{LL}^{-1} m_{LR}$. We choose a basis U_R where $M_{RR}^{diag} = U_R^T M_{RR} U_R = \text{diag}(M_1, M_2, M_3)$ with real and positive eigenvalues. We then transform diagonal form of Dirac mass matrix, $m_{LR} = \text{diag}(\lambda^m, \lambda^n, 1)v$ to the U_R basis: $m_{LR} \rightarrow m'_{LR} = m_{LR} U_R Q$ where $Q = \text{diag}(1, e^{i\alpha}, e^{i\beta})$ is the complex matrix containing CP-violating Majorana phases derived from M_{RR} . Here λ is the Wolfenstein parameter and the choice (m, n) in m_{LR} gives the type of Dirac mass matrix. For example, $(6, 2)$ for charged-lepton type mass matrix and $(8, 4)$ for up-quark type mass matrix. In this prime basis the Dirac neutrino Yukawa coupling becomes $h' = \frac{m'_{LR}}{v}$ which enters in the expression of CP-asymmetry ϵ_1 . The Yukawa coupling matrix h' also becomes complex, and hence the term $\text{Im}(h^\dagger h)_{1j}$ appearing in lepton asymmetry ϵ_1 gives a non-zero contribution. A straightforward simplification shows that $(h^\dagger h)_{1j}^2 = (Q_{11}^*)^2 Q_{22}^2 R_2 + (Q_{11}^*)^2 Q_{33}^2 R_3$ where $R_{2,3}$ are real parameters. After inserting the values of phases the above expression leads to $\text{Im}(h^\dagger h)_{1j}^2 = -[R_2 \sin 2(\alpha - \beta) + R_3 \sin 2\alpha]$ which imparts non-zero CP asymmetry for particular choice of (α, β) .

In our numerical estimation of lepton asymmetry, we choose some arbitrary values of α and β other than $\pi/2$ and 0. For example, light neutrino masses $(m_1, -m_2, m_3)$ lead to $M_{RR}^{diag} = \text{diag}(M_1, -M_2, M_3)$, and we thus fix the Majorana phase $Q = \text{diag}(1, e^{i\alpha}, e^{i\beta}) = \text{diag}(1, e^{i(\pi/2+\pi/4)}, e^{i\pi/4})$ for

Type	$\Delta m_{21}^2 [10^{-5} eV^2]$	$\Delta m_{23}^2 [10^{-3} eV^2]$	$\tan^2 \theta_{12}$	$\sin^2 2\theta_{23}$	$\sin \theta_{13}$
Deg.(IA)	7.8	2.6	0.5	1.0	0.0
Deg.(IB)	7.9	2.5	0.5	1.0	0.0
Deg.(IC)	7.9	2.5	0.5	1.0	0.0
Inh.(IIA)	7.3	2.5	0.5	1.0	0.0
Inh.(IIB)	8.5	2.3	0.5	1.0	0.0
Nh.(IIIA)	7.1	2.1	0.5	1.0	0.0
Nh.(IIIB)	7.5	2.4	0.5	1.0	0.0

Table 1: Predicted values of the solar and atmospheric neutrino mass-squared differences for $\tan^2 \theta_{12}=0.50$, using m_{LL} given in the Appendix A.

$\alpha = (\pi/4 + \pi/2)$ and $\beta = \pi/4$. The extra phase $\pi/2$ in α absorbs the negative sign before heavy Majorana mass M_2 . In our search programme such choice of the phases leads to highest numerical estimations of lepton CP asymmetry.

In Table 1 we give the predictions on Δm_{21}^2 and Δm_{23}^2 of these seven neutrino mass models under consideration in Appendix A. They obey $\mu - \tau$ symmetry and predict tribimaximal mixings in addition. In Table 2 the three heavy right-handed neutrino masses are extracted from the right-handed Majorana mass matrices so constructed through inverse seesaw formula, for three choices of diagonal Dirac neutrino mass matrices. We get degenerate spectrum of heavy Majorana masses for normal hierarchical model and this allows us to use resonant leptogenesis formula. The corresponding baryon asymmetry Y_B are estimated in Table 3 and this shows that only normal hierarchical model predict reasonable values whereas inverted hierarchical model (IIB) nearly misses the observational bound. Degenerate models predict too low baryon asymmetry.

Our estimated baryon asymmetry for normal hierarchical model(IIIA, IIIB) lies between 9.27×10^{-9} with Dirac neutrino mass matrix as charged lepton mass matrix $(6, 2)$, and 7.28×10^{-11} for the up-quark mass matrix $(8, 4)$. This hints a possible choice of Dirac neutrino mass matrix lying between these two e.g., $m_{LR} = \text{diag.}(\lambda^8, \lambda^2, 1)v$. As emphasised earlier, our starting point is the neutrino mass matrix which satisfies the observed neutrino mass parameters and mixings. The values of input parameters are fixed at this level before applying to the calculation of baryon asymmetry. The whole calculation is performed in a consistent way.

Type	(m,n)	M_1	M_2	M_3
IA	(6,2)	1.22×10^8	-6.01×10^{11}	2.59×10^{13}
IA	(8,4)	9.86×10^5	-5.03×10^9	2.51×10^{13}
IB	(6,2)	4.05×10^7	6.16×10^{11}	7.60×10^{13}
IB	(8,4)	3.28×10^5	4.99×10^9	7.60×10^{13}
IC	(6,2)	4.05×10^7	-6.69×10^{12}	6.99×10^{12}
IC	(8,4)	3.28×10^5	-4.83×10^{11}	7.84×10^{11}
IIA	(6,2)	3.29×10^8	9.73×10^{12}	6.25×10^{16}
IIA	(8,4)	2.63×10^6	7.94×10^{10}	6.21×10^{16}
IIB	(6,2)	-9.97×10^8	2.63×10^{12}	5.59×10^{14}
IIB	(8,4)	-8.10×10^6	2.14×10^{10}	5.57×10^{14}
IIIA	(6,2)	3.93×10^{11}	-4.09×10^{11}	2.87×10^{14}
IIIA	(8,4)	3.19×10^9	-3.22×10^9	2.85×10^{14}
IIIB	(6,2)	3.85×10^{11}	-3.99×10^{11}	2.99×10^{14}
IIIB	(8,4)	3.13×10^9	-3.25×10^9	2.97×10^{14}

Table 2: Heavy right-handed Majorana neutrino masses M_j for degenerate models (IA,IB,IC), inverted models (IIA,IIB) and normal hierarchical models (IIIA, IIIB), with $\tan^2 \theta_{12}=0.5$, using neutrino mass matrices given in Appendix A. The entry (m, n) in m_{LR} , indicates the type of Dirac neutrino mass matrix taken as charged lepton mass matrix (6,2) or up quark mass matrix (8,4), as explained in the text.

Type	(m,n)	$\tilde{m}_1(GeV)$	$(h^\dagger h)_{11}$	k_1	ϵ_1	Y_B
IA	(6,2)	1.19×10^{-9}	4.78×10^{-6}	9.3×10^{-5}	1.53×10^{-7}	1.55×10^{-13}
IA	(8,4)	1.19×10^{-9}	3.87×10^{-8}	9.3×10^{-5}	4.14×10^{-9}	4.16×10^{-15}
IB	(6,2)	3.97×10^{-10}	5.31×10^{-7}	2.83×10^{-4}	4.46×10^{-16}	1.36×10^{-21}
IB	(8,4)	3.97×10^{-10}	4.30×10^{-9}	2.83×10^{-4}	3.62×10^{-18}	1.10×10^{-23}
IC	(4,2)	3.97×10^{-10}	5.31×10^{-7}	2.83×10^{-4}	2.49×10^{-15}	7.62×10^{-21}
IC	(8,4)	3.97×10^{-10}	4.30×10^{-9}	2.83×10^{-4}	2.16×10^{-16}	6.62×10^{-22}
IIA	(6,2)	4.95×10^{-11}	5.31×10^{-7}	2.95×10^{-3}	1.56×10^{-12}	4.98×10^{-17}
IIA	(8,4)	4.95×10^{-11}	4.30×10^{-9}	2.95×10^{-3}	1.26×10^{-14}	4.04×10^{-19}
IIB	(6,2)	1.08×10^{-12}	5.01×10^{-6}	8.83×10^{-4}	2.69×10^{-7}	2.57×10^{-12}
IIB	(8,4)	1.52×10^{-10}	4.06×10^{-8}	8.83×10^{-4}	2.18×10^{-9}	2.07×10^{-14}
IIIA	(6,2)	5.80×10^{-10}	7.51×10^{-3}	1.82×10^{-4}	4.59×10^{-3}	9.27×10^{-9}
IIIA	(8,4)	5.80×10^{-10}	6.13×10^{-5}	1.82×10^{-4}	3.62×10^{-5}	7.28×10^{-11}
IIIB	(6,2)	5.93×10^{-10}	7.51×10^{-3}	1.83×10^{-4}	4.91×10^{-3}	9.66×10^{-9}
IIIB	(8,4)	5.93×10^{-10}	6.13×10^{-5}	1.83×10^{-4}	3.88×10^{-5}	7.59×10^{-11}

Table 3: Values of CP asymmetry and the baryon asymmetry for degenerate models (IA, IB, IC), inverted hierarchical models (IIA, IIB) and normal hierarchical models (IIIA, IIIB) with for $\tan^2 \theta_{12} = 0.50$, using mass matrices given in Appendix A. The entry (m, n) indicates the type of Dirac mass matrix as explained in the text.

Type	(m,n)	$T_R^{min} < T_R \leq T_R^{max}$ (GeV)	$M_I^{min} < M_I \leq M_I^{max}$ (GeV)
IA	(6,2)	$2.61 \times 10^5 < T_R \leq 1.22 \times 10^6$	$2.44 \times 10^8 < M_I \leq 1.15 \times 10^9$
IA	(8,4)	$7.80 \times 10^4 < T_R \leq 9.86 \times 10^3$	$1.97 \times 10^6 < M_I \leq 2.49 \times 10^5$
IB	(6,2)	$2.97 \times 10^{13} < T_R \leq 4.05 \times 10^5$	$9.10 \times 10^7 < M_I \leq 0.49$
IB	(8,4)	$2.97 \times 10^{13} < T_R \leq 3.28 \times 10^3$	$6.56 \times 10^5 < M_I \leq 7.20 \times 10^{-5}$
IC	(6,2)	$5.29 \times 10^{13} < T_R \leq 4.05 \times 10^5$	$8.05 \times 10^7 < M_I \leq 2.74$
IC	(8,4)	$4.97 \times 10^{11} < T_R \leq 3.28 \times 10^3$	$6.56 \times 10^5 < M_I \leq 4.35 \times 10^{-3}$
IIA	(6,2)	$6.80 \times 10^{10} < T_R \leq 3.25 \times 10^6$	$6.50 \times 10^8 < M_I \leq 3.09 \times 10^4$
IIA	(8,4)	$6.80 \times 10^{10} < T_R \leq 2.64 \times 10^4$	$5.26 \times 10^6 < M_I \leq 2.03$
IIB	(6,2)	$1.22 \times 10^6 < T_R \leq 9.99 \times 10^6$	$1.99 \times 10^9 < M_I \leq 1.64 \times 10^{10}$
IIB	(8,4)	$1.22 \times 10^6 < T_R \leq 8.1 \times 10^4$	$1.62 \times 10^7 < M_I \leq 1.08 \times 10^6$
IIIA	(6,2)	$2.80 \times 10^5 < T_R \leq 3.93 \times 10^9$	$7.86 \times 10^{11} < M_I \leq 1.10 \times 10^{17}$
IIIA	(8,4)	$2.88 \times 10^5 < T_R \leq 3.19 \times 10^7$	$6.38 \times 10^9 < M_I \leq 7.06 \times 10^{12}$
IIIB	(6,2)	$2.57 \times 10^4 < T_R \leq 3.85 \times 10^9$	$7.70 \times 10^{11} < M_I \leq 1.15 \times 10^{17}$
IIIB	(8,4)	$2.64 \times 10^4 < T_R \leq 3.13 \times 10^7$	$6.26 \times 10^9 < M_I \leq 7.42 \times 10^{12}$

Table 4: Theoretical bounds on reheating temperature T_R and inflaton mass M_I in non-thermal leptogenesis, for all neutrino mass models described in Tables 1-3.

3 Non-thermal leptogenesis

We next apply the neutrino mass models discussed in section 2 (Tables 1-3) to non-thermal leptogenesis scenario [7] where the right-handed neutrinos are produced through the direct non-thermal decay of the inflaton. We follow the standard procedure outlined in ref. [20] where non-thermal leptogenesis and baryon asymmetry in the universe had been studied in different neutrino mass models whereby some mass models were excluded using bounds from below and from above on the inflation mass and reheating temperature after inflation. Though we adopt similar analysis, the texture of the neutrino mass models considered here are different and hence the conclusions are also expected to be different.

We start with the inflation decay rate given by

$$\Gamma_\phi = \Gamma(\phi \rightarrow N_i N_i) \simeq \frac{|\lambda_i|^2}{4\pi} M_I \quad (10)$$

where λ_i are the Yukawa coupling constants for the interaction of three heavy right-handed neutrinos N_i with the inflaton ϕ of mass M_I . The reheating

temperature after inflation is given by the expression,

$$T_R = \left(\frac{45}{2\pi^2 g_*} \right)^{1/4} (\Gamma_\phi M_P)^{1/2} \quad (11)$$

where $M_P \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass[21] and g_* is the effective number of relativistic degrees of freedom at reheating temperature. For SM we have $g_* = 106.75$ and for MSSM $g_* = 228.75$. If the inflaton dominantly couples to N , the branching ratio of this decay process is taken as $BR \sim 1$, and the produced baryon asymmetry of the universe can be calculated by the following relation [22],

$$Y_B = \frac{n_B}{s} = CY_L = C \frac{3}{2} \frac{T_R}{M_I} \epsilon \quad (12)$$

where Y_L is the lepton asymmetry generated by CP-violating out-of-equilibrium decays of heavy neutrino N_1 and T_R is the reheating temperature. The fraction C has the value $C = -28/79$ for SM and $C = -8/15$ in the MSSM.

The above expression (12) of the baryon asymmetry is supplemented by two more boundary conditions [20]: (i) lower bound on inflaton mass $M_I > 2M_1$ coming from allowed kinematics of inflaton decay, and (ii) an upper bound for the reheating temperature $T_R \leq 0.01M_1$ coming from out-of-thermal equilibrium decay of N_1 . Using the observed central value[2] of the baryon asymmetry $Y_B = \frac{n_B}{s} = 8.7 \times 10^{-11}$ and theoretical prediction of CP asymmetry ϵ in Table 3, in equation (12), one can establish the relation between T_R and M_I for each neutrino mass model.

The right-handed neutrino mass M_1 from Table 2 and the CP asymmetry ϵ from Table 3 for all neutrino mass models, are used to calculate the bounds: $T_R^{min} < T_R \leq T_R^{max}$ and $M_I^{min} < M_I \leq M_I^{max}$ in Table 4 following eq.(12) along with other two boundary conditions cited above. Only those models which satisfy the constraint $T_R^{max} > T_R^{min}$ could survive in the non-thermal leptogenesis. These models are identified as IA with (6,2), IIB with (6,2), III (A,B) with (6,2) and III(A,B) with (8,4) where (m,n) refers to the type of Dirac neutrino mass matrix. From Table 4 it is seen that inflationary models in which $M_I \sim 10^{13}$ GeV, like e.g., chaotic or natural inflation, are compatible only with normal hierarchical model III (A, B) with (6,2). In fact with $T_R = 10^6$ GeV, we get $M_I = 2.8 \times 10^{13}$ GeV., $\Gamma_\phi = 2.85 \times 10^{-6}$ GeV, and $|\lambda_1| = 1.13 \times 10^{-8}$ which are compatible with chaotic inflationary model.

In supersymmetric models, the gravitino problem [8] can be avoided provided that the reheating temperature after inflation is bounded from above

in a certain way, namely $T_R \leq (10^6 - 10^7)$ GeV. In fact the reheating temperature $T_R = 10^6$ GeV is relevant in order to realise the weak scale gravitino mass $m_{3/2} \sim 100$ GeV without causing the gravitino problem. Even this reheating temperature is relaxed for two order $T_R = 10^7$ GeV, we would have $M_I \sim 10^{11}$ GeV in normal hierarchy type III(A,B) with (8,4). We conclude that the only surviving model in this analysis is the normal hierarchical model (III).

4 Summary and discussion

To summarise, we first parametrise the light left-handed Majorana neutrino mass matrices describing the possible patterns of neutrino masses viz, degenerate, inverted hierarchical and normal hierarchical, which obey the $\mu - \tau$ symmetry having tribimaximal mixings. As a first test these mass matrices predict the neutrino mass parameters and mixings consistent with data, and all the input parameters are fixed at this stage. In the next stage these mass matrices are employed to estimate the baryon asymmetry in both thermal as well as non-thermal leptogenesis scenario. We use the CP violating Majorana phases derived from right-handed Majorana mass matrix and two possible forms of Dirac neutrino mass matrices as either charged lepton mass matrix or up-quark mass matrix in the calculation. The overall analysis shows that normal hierarchical model appears to be the most favourable choice in nature. The present analysis though phenomenological may serve as an additional criteria to discard some of the presently available neutrino mass models and neutrino mass ordering patterns. There are some suggestions in the literature[24] for inverted hierarchical model to enhance the estimation of baryon asymmetry if m_3 is increased. The present investigation has taken care of the maximum allowed non-zero value of $m_3 \sim 0.033$ eV in case of inverted hierarchy type IIB model. Our result also differs from a recent study in nonthermal leptogenesis with strongly hierarchical right-handed neutrinos[25] where the mass of the lightest right handed neutrino $M_1 \leq 10^6$ GeV. There are some propositions[26] for probing the reheating temperature at the Large Hadron Collider and this hopefully decides the validity of thermal leptogenesis.

Appendix A

Classification:

We first list here for ready reference to the classification of neutrino mass models, the zeroth-order left-handed Majorana neutrino mass matrices with texture zeros, m_{LL} , corresponding to three models of neutrinos given in the text, viz., degenerate (Type [I]), inverted hierarchical (Type [II]) and normal hierarchical (Type [III]).

Type	m_{LL}	m_{LL}^{diag}
[IA]	$\begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} m_0$	$Diag(1, -1, 1)m_0$
[IB]	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_0$	$Diag(1, 1, 1)m_0$
[IC]	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} m_0$	$Diag(1, 1, -1)m_0$
[IIA]	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} m_0$	$Diag(1, 1, 0)m_0$
[IIB]	$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} m_0$	$Diag(1, -1, 0)m_0$
[III]	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} m_0$	$Diag(0, 0, 1)m_0$

Parametrisation with two parameters for tribimaximal mixings:

Left-handed Majorana neutrino mass matrices which obey $\mu - \tau$ symmetry[10,23] have the following form

$$m_{LL} = \begin{pmatrix} X & Y & Y \\ Y & Z & W \\ Y & W & Z \end{pmatrix} m_o$$

This predicts an arbitrary solar mixing angle $\tan 2\theta_{12} = |\frac{2\sqrt{2}Y}{(X-Z-W)}|$, while the predictions on atmospheric mixing angle is maximal ($\theta_{23} = \pi/4$) and Chooz angle zero. We parametrise the mass matrices (with only two parameters) whereby the solar mixing is fixed at tribimaximal mixings for all possible patterns of neutrino mass models:

1.Deg Type A [IA]($m_i = m_1, -m_2, m_3$)

$$m_{LL} = \begin{pmatrix} \delta_1 - 2\delta_2 & -\delta_1 & -\delta_1 \\ -\delta_1 & \frac{1}{2} - \delta_2 & -\frac{1}{2} - \delta_2 \\ -\delta_1 & -\frac{1}{2} - \delta_2 & \frac{1}{2} - \delta_2 \end{pmatrix} m_o$$

with input values: $\delta_1=0.66115, \delta_2=0.16535, m_o = 0.4eV$.

2.Deg Type B [IB]($m_i = m_1, m_2, m_3$)

$$m_{LL} = \begin{pmatrix} 1 - \delta_1 - 2\delta_2 & \delta_1 & \delta_1 \\ \delta_1 & 1 - \delta_2 & -\delta_2 \\ \delta_1 & -\delta_2 & 1 - \delta_2 \end{pmatrix} m_o$$

with input values: $\delta_1=8.314 \times 10^{-5}, \delta_2=0.00395, m_o=0.4eV$.

3.Deg Type C [IC]($m_i = m_1, m_2, -m_3$)

$$m_{LL} = \begin{pmatrix} 1 - \delta_1 - 2\delta_2 & \delta_1 & \delta_1 \\ \delta_1 & -\delta_2 & 1 - \delta_2 \\ \delta_1 & 1 - \delta_2 & -\delta_2 \end{pmatrix} m_o$$

with input values: $\delta_1=8.314 \times 10^{-5}, \delta_2=0.00395, m_o=0.4eV$.

4: Inverted Hierarchical mass matrix **with** $m_3 \neq 0$:

$$m_{LL}(IH) = \begin{pmatrix} 1 - 2\epsilon & -\epsilon & -\epsilon \\ -\epsilon & 1/2 & 1/2 - \eta \\ -\epsilon & 1/2 - \eta & 1/2 \end{pmatrix} m_0.$$

Inverted Hierarchy with even CP parity in the first two mass eigenvalues [IIA] ($m_1 = m_1, m_2, m_3$): $\eta/\epsilon=1.0, \eta=0.0048, m_0 = 0.05eV$.
 Inverted Hierarchy with odd CP parity in the first two mass eigenvalues [IIB] ($m_i = m_1, -m_2, m_3$): $\eta/\epsilon=1.0, \eta=0.6607, m_0 = 0.05eV$.

5: Normal Hierarchical mass matrix **Case (i) with** $m(1,1) \neq 0$ type-[IIIA]:

$$m_{LL}(NH) = \begin{pmatrix} -\eta & -\epsilon & -\epsilon \\ -\epsilon & 1 - \epsilon & -1 \\ -\epsilon & -1 & 1 - \epsilon \end{pmatrix} m_0$$

with input values: $\eta/\epsilon=0.0, \epsilon=0.175, m_0 = 0.029eV$.

6: Normal Hierarchical mass matrix **Case (ii) with** $m(1,1) = 0$ type-[IIIB]:

$$m_{LL}(NH) = \begin{pmatrix} 0 & -\epsilon & -\epsilon \\ -\epsilon & 1 - \epsilon & -1 + \eta \\ -\epsilon & -1 + \eta & 1 - \epsilon \end{pmatrix} m_0$$

with input values: $\eta/\epsilon=0.0, \epsilon=0.164, m_0 = 0.028eV$.

The textures of mass matrices for inverted hierarchy (IIA, IIB) as well as normal hierarchy (IIIA, IIIB) have the potential to decrease the solar mixing angle from the tribimaximal value, without sacrificing $\mu - \tau$ symmetry. This is possible through the identification of 'flavour twister' $\eta/\epsilon \neq 0$ [23].

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